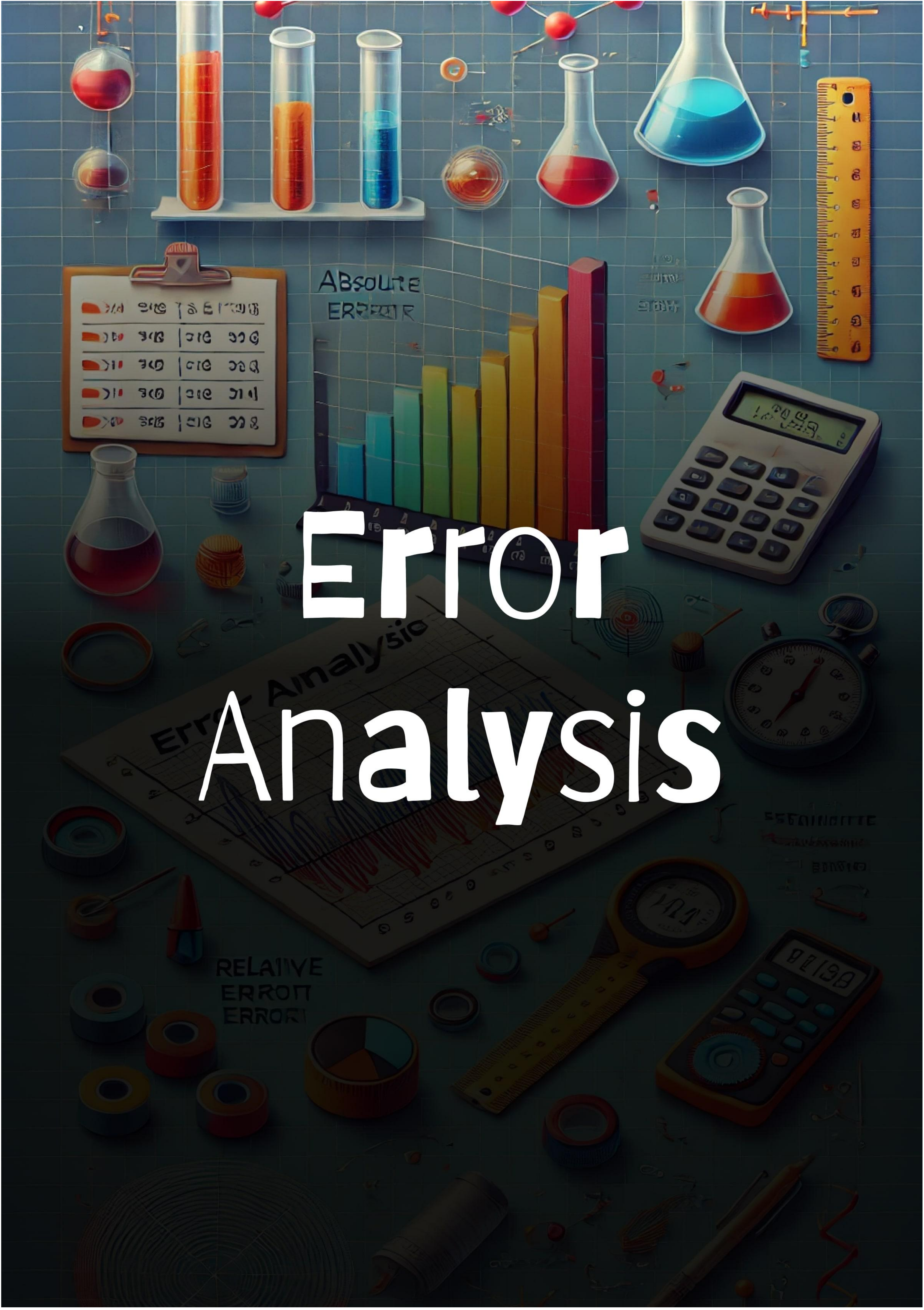

Error Analysis

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Error Analysis

PROPAGATION OF ERRORS

IN ADDITION

Let two physical quantities be given as

$$p_1 = P_1 \pm \Delta P_1$$

$$\& \quad p_2 = P_2 \pm \Delta P_2 \quad \text{, then}$$

in the sum $(p_1 + p_2)$, the errors get added

i.e.

$$P_1 + P_2 = P_1 + P_2 \pm (\Delta P_1 + \Delta P_2)$$

★ all operations should be done as per rules of significant fig.

Eg: $R_1 = 10.0 \pm 0.1\,\Omega$

$R_2 = 10.7 \pm 0.1\,\Omega$

$R_{eff} = 20.7 \pm 0.2\,\Omega$

IN SUBTRACTION

Let two quantities be given as

$$p_1 = P_1 \pm \Delta P_1$$

$$p_2 = P_2 \pm \Delta P_2 \quad \text{, then}$$

their difference $(p_1 - p_2)$ is given by

$$p_1 - p_2 = P_1 - P_2 \pm (\Delta P_1 + \Delta P_2)$$

IN MULTIPLICATION

Case I: When relative errors are small

Relative error in product is sum of relative errors of terms being multiplied.

$$p_1 = P_1 \pm \Delta P_1 \quad \text{where } \Delta P_1 << P_1$$

$$p_2 = P_2 \pm \Delta P_2 \quad \text{where } \Delta P_2 << P_2$$

$$q = p_1 p_2$$

$$\ln q = \ln p_1 + \ln p_2$$

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$$\frac{dq}{q} = \frac{dp_1}{p_1} + \frac{dp_2}{p_2}$$

$$\frac{\Delta q}{q} \simeq \frac{\Delta P_1}{P_1} + \frac{\Delta P_2}{P_2}$$

$$p_1 p_2 = q \pm \Delta q$$

Case II : When relative errors are large.

(Say >10%) then we need to physically find out the upper and lower bound of the product & report the answer accordingly.

Eg - $V = 10V \pm 10\%$

$I = 5A \pm 10\%$

$$P_{up} = V_u I_u = (11)(5.5) = 60.5W$$

$$P_{Lo} = V_L P_L = (9)(4.5) = 40.5W$$

$$P = 50.5 \pm 10\ W$$

IN DIVISION

Relative errors get added if relative errors are small.

$$q = \frac{p_1}{p_2}$$

$$\ln q = \ln p_1 - \ln p_2$$

$$\frac{dq}{q} = \frac{dp_1}{p_1} - \frac{dp_2}{p_2}$$

$$\frac{\Delta q}{q} \simeq \frac{\Delta p_1}{p_1} + \frac{\Delta p_2}{p_2}$$

However, in case relative errors are large, we will once again need to find upper & lower bounds.

Eg - $V = 10V \pm 10\%$

$I = 5A \pm 10\%$

$R = 2\Omega \pm 20\%$

$= 2 \pm 0.4\Omega$

$R_u = \frac{11}{4.5}$, $R_L = \frac{9}{5.5}$

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$$R = \frac{1}{2}\left(\frac{11}{4.5}+\frac{9}{5.5}\right) \pm \frac{1}{2}\left(\frac{11}{4.5}-\frac{9}{5.5}\right)$$

$$= 2.035 \pm 0.405\,\Omega$$

IN MULTIPLICATION OF POWERS / SURDS EXPRESSIONS

Here, we estimate the max relative error by taking log & differentiating.

Que.) Find relative error in T in terms of relative error in l and g.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\ln(T) = \frac{1}{2}\ln l - \frac{1}{2}\ln g$$

$$\frac{dT}{T} = \frac{dl}{2l} - \frac{dg}{2g}$$

$$\left|\frac{dT}{T}\right|_{max} = \frac{1}{2}\left|\frac{dl}{l}\right| + \frac{1}{2}\left|\frac{dg}{g}\right|$$

$$\frac{\Delta T}{T} \simeq \frac{1}{2}\frac{\Delta l}{l} + \frac{1}{2}\frac{\Delta g}{g}$$

Que.) $P = \frac{A^a}{B^b}$

$$\frac{\Delta P}{P} \simeq a\frac{\Delta A}{A} + b\frac{\Delta B}{B}$$

Que.) length of pendulum = $1 \pm 0.1\,m$
& $g = 9.8 \pm 0.1\,m/s^2$
Find T of a pendulum. $\left(T_0 = 2\pi\sqrt{\frac{1}{9.8}}\right)$

$$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{0.1}{1}\right) + \frac{1}{2}\left(\frac{0.1}{9.8}\right)$$

$$\frac{\Delta T}{T} = \frac{1}{2}(0.1)\left[1+\frac{1}{9.8}\right]$$

$$= \frac{0.1}{2}\left[\frac{9.9}{9.8}\right]$$

$$\frac{\Delta T}{T}\times 100 = \frac{1}{2}(10) + \frac{1}{2}(1) \simeq 5.5$$

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$T = 2.077 \pm 5.5\% \rightarrow 0.11039$

$T \simeq 2.0 \pm 0.1$

IN ARBITRARY FUNCTIONS

There is no full proof method for finding errors in general functions as some functions are so sensitive that a little change in one quantity can lead to an enormous change in the value of the function.

However, for most well-behaved function, with reasonably small errors in their parameters, we can find the error in the function as

If $W = f(x, y, z)$

then $\Delta W \simeq \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \frac{\partial f}{\partial z} \Delta z$

Eg: $y = \sin(x \pm \Delta x)$

$y = \sin x \pm \frac{\partial}{\partial x} \sin(x) . \Delta x$

$y = \sin x \pm \cos(x) . \Delta x$

$y = f(x \pm \delta x)$

then $y = (\pm \delta x \, f'(x)) + f(x)$

Eg: $\sin 32^\circ = \underline{0.5299}$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} \left(\cancel{2} . \frac{\pi}{\cancel{180}_{90}}\right)$

$= \frac{1}{2}\left(1 + \frac{\sqrt{3}\pi}{90}\right)$

$= \underline{0.53023}$

DIMENSIONS

Fundamental quantities which can be used to express all other physical quantities.

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CGS units	Fundamental Dimensions	SI Unit
g.	Mass	Kg.
cm.	length	m.
Sec.	Time	Sec.
K.	$\theta \to$ Temp.	Kelvin
Cd.	Cd $\to$ Candela (luminous Intensity)	Candela
	A $\to$ Current	ampere
	n $\to$ moles / quantity of matter	

- Velocity LT^{-1}
- Acceleration LT^{-2}
- Force MLT^{-2}
- Energy ML^2T^{-2}
- Current A
- Momentum MLT^{-1}
- Power ML^2T^{-3}
- Planck's Constant ML^2T^{-1} $\left(\frac{E\lambda}{c} = \frac{ML^2T^{-2}\cdot L}{LT^{-1}}\right)$

Que.) Find dimensions:

→ Electric Potential → $ML^2T^{-3}A^{-1}$

→ Electric Field → $MLT^{-3}A^{-1}$

→ Resistance → $ML^2T^{-3}A^{-2}$

→ μ_0 → $\left(\frac{q}{EA}\right)\left(\frac{AT}{MLT^{-3}A^{-1}L^2}\right)$

$= M^1L^{-3}A^2T^4$

→ Inductance → $\left(\frac{\text{Energy}}{i^2}\right) = ML^2T^{-2}A^{-2}$

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→ Coefficient of viscosity (η) $\left(\frac{F}{\alpha v}\right)$ $\left(\frac{MLT^{-2}}{LLT^{-1}}\right)$ $= ML^{-1}T^{-1}$

Que.) If force (F), velocity (v), acceleration (a) were to be used as fundamental dimensions, What will be dimensions of M, L, T.

$\boxed{M} = F^{x} v^{y} a^{z}$

$M = (MLT^{-2})^{x} (LT^{-1})^{y} (LT^{-2})^{z}$

$x = 1, \quad x+y+z=0$

$y+z=-1$

$-2x-y-2z=0$

$y+2z=-2$

$z=-1$

$y=0$

$M = F^{1}a^{-1}$

$\boxed{L} = x=0 \quad , \; y+z=1$

$-y-2z=0$

$y=z \quad , \; z=-1 \quad , \; y=-2z$

$L = v^{2}a^{-1}$

$\boxed{T} \quad x=0 \quad , \; y+z=0$

$-y-2z=1$

$y=1 \; , \; z=-1$

$T = Va^{-1}$

Zero Correction = - (Zero error)

actual value - shown value . shown value - actual value

Que.) T depends on (I, M, g, d)

I → moment of inertia

d → distance

$T = K[I]^{x}[M]^{y}[g]^{z}[d]^{w}$

$T = [ML^{2}]^{x}[M]^{y}[LT^{2}]^{z}[L]^{w}$

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$x+y=0 \quad , \quad 2x+z+w=0$

$-2z=1$

$z=-1/2$

$x=-y \quad , \quad -2y-\frac{1}{2}+w=0$

$w-2y=\frac{1}{2}$

$x=-\frac{1}{2}(w-\frac{1}{2})$

$w=2y+\frac{1}{2}$

$y=\frac{5}{2}(w-1)$

Que.) T depends on (I, Mg, d)

$T = K[ML^2]^x [MLT^{-2}]^y [L]^w$

$x+y=0$

$x=-y$

$x=\frac{1}{2}$

$2x+y+w=0$

$1-\frac{1}{2}+w=0$

$w=-\frac{1}{2}$

$-2y=1$

$y=-\frac{1}{2}$

$T = K(I)^{1/2}(Mg)^{-1/2}(d)^{-1/2}$

$T = K\sqrt{\frac{I}{Mgd}}$

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